

Super 4PCS



Fast Global Pointcloud Registration via Smart Indexing



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² Google Inc.

Problem Statement

- Estimate rigid transformation \mathbf{tr}
 - Large search space (6DoF)



P

+



Q

=



P + tr(Q)

Problem Statement

- Estimate rigid transformation \mathbf{tr}
 - **Local** registration: from an input pose
 - ICP [BM92], [CM92], [RL01], [MGPG04]
 - Sparse ICP [BTP13]
 - Kinect Fusion [IKH*11]



$P + \mathbf{tr}(Q)$

Problem Statement

- Estimate rigid transformation \mathbf{tr}
 - **Local** registration: from an input pose
 - ICP [BM92], [CM92], [RL01], [MGPG04]
 - Sparse ICP [BTP13]
 - Kinect Fusion [IKH*11]
 - **Global** registration: arbitrary input pose
 - RANSAC [FB81], [IR96], [CH99]
 - and variants [GMGP05], [PB09], [PB11], [ART10], [RABT13]
 - 4 Point Congruent Set [AMCO08]

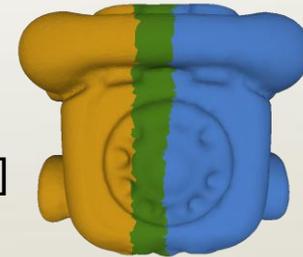
3 pairs of corresponding points are sufficient to define a rigid transformation

Problem Statement

RANSAC: $O(n^3)$

4 Point Congruent Set (4PCS) : $O(n^2)$

Difficult cases



[T13]



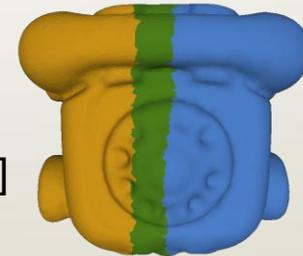
Problem Statement

RANSAC: $O(n^3)$

4 Point Congruent Set (4PCS) : $O(n^2)$

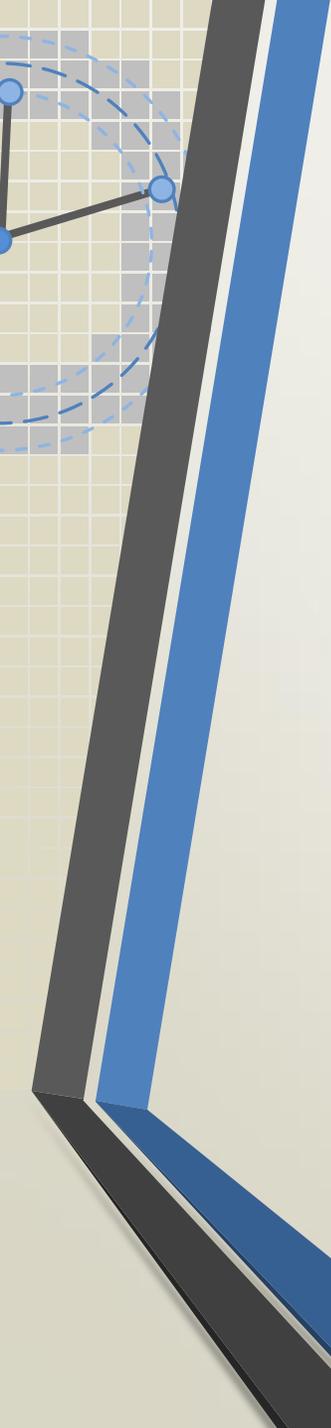
Our approach (Super 4PCS): $O(n)$

Difficult cases



[T13]





```
~/super4pcs/code
File Edit View Search Terminal Help
~/git/super4pcs/code$ ./4pcs
Use Super4PCS
Work with 226 points
norm_max_dist: 5.000000
Initial LCP: 0.061947
Computation time (sec): 9.996068
Score: 0.45132744
0.451327
(Homogeneous) Transformation from input2.obj to input1.obj:

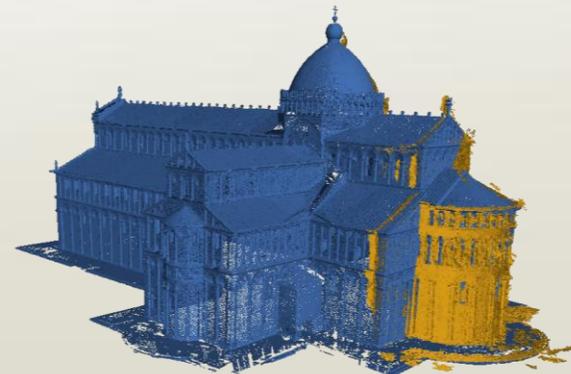
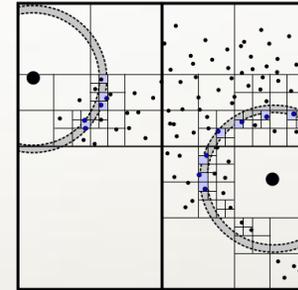
          0.978          -0.171          -0.118          90.768
          0.071           0.808          -0.585         386.750
          0.195           0.564           0.803          93.319
          0.000           0.000           0.000           1.000

saving transform matrix to output_transform.mx
Merged object was written to output.obj
~/git/super4pcs/code$
```

Demo

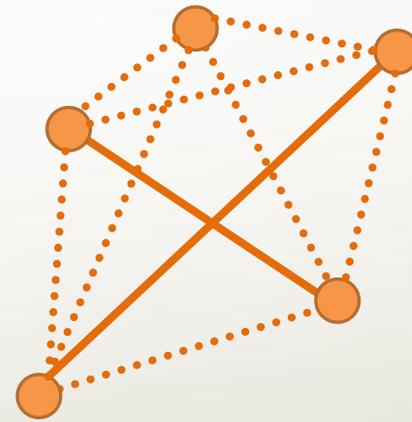
Overview

- 4PCS
- Super4PCS
- Results

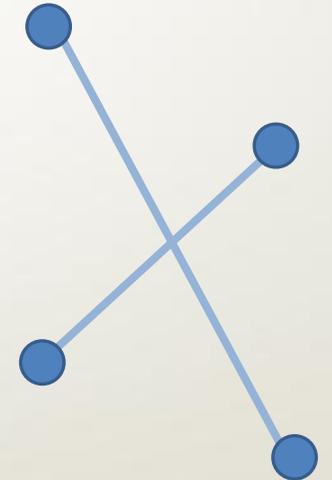


4 Point Congruent Set

- Use planar 4-points basis in P
- Find congruent 4-points in Q



Q



P

4-points Congruent Sets for Robust Surface Registration

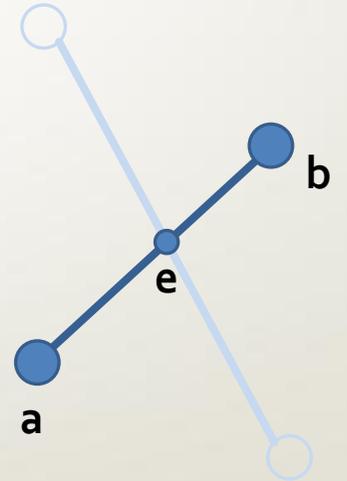
Dror Aiger, Niloy J. Mitra, Daniel Cohen-Or

SIGGRAPH 2008

4 Point Congruent Set

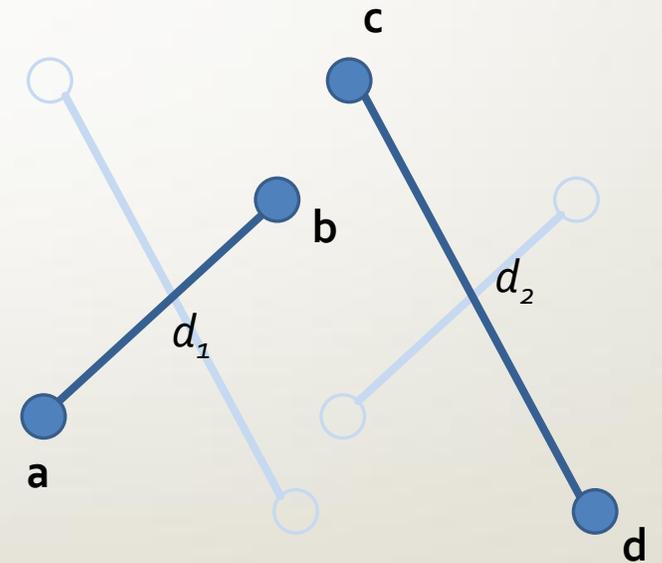
- What does congruent mean ?
- Similar under a given transformation class
 - Ratios r_1 and r_2

$$r_1 = \| \mathbf{a} - \mathbf{e} \| / \| \mathbf{a} - \mathbf{b} \|$$



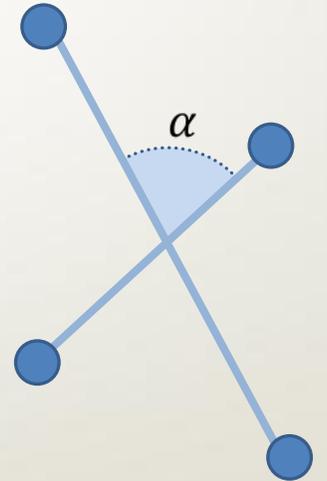
4 Point Congruent Set

- What does congruent mean ?
- Similar under a given transformation class
 - Ratios r_1 and r_2
 - Distances d_1, d_2



4 Point Congruent Set

- What does congruent mean ?
 - Similar under a given transformation class
 - Ratios r_1 and r_2
 - Distances d_1, d_2
 - Angle α
- } Rigid transformation



4 Point Congruent Set

Select Base

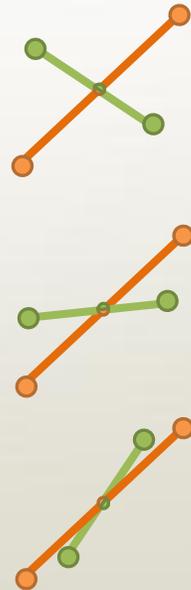


Extract pairs



Extract congruent superset

Ratios
Distances



Filter congruent set

Ratios
Distances
Angles



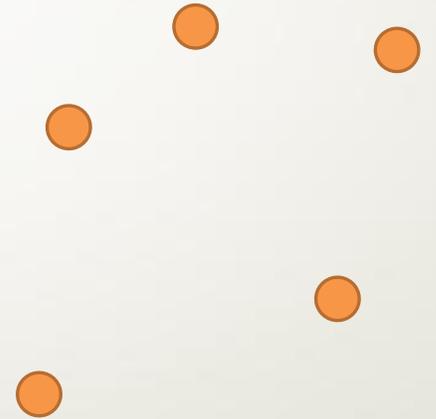
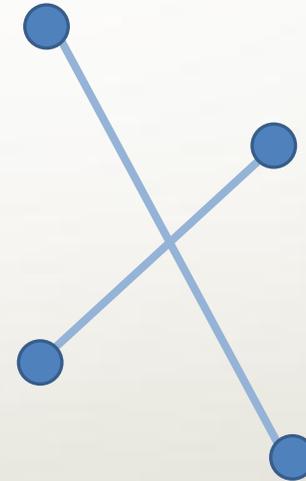
Verify and update transformation



False positives (non congruents)

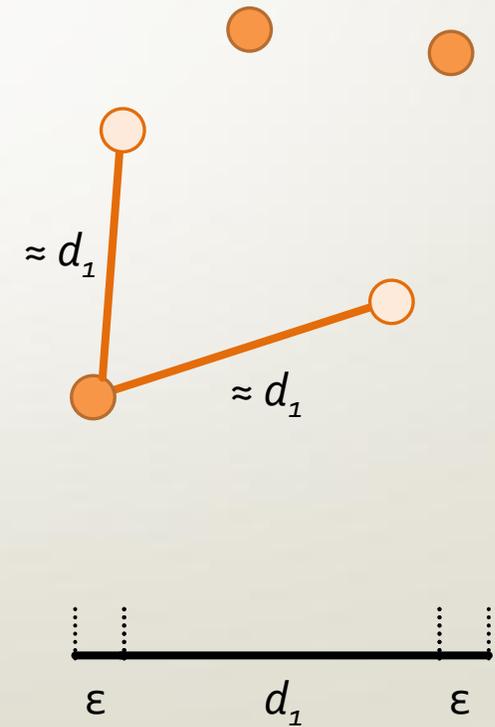
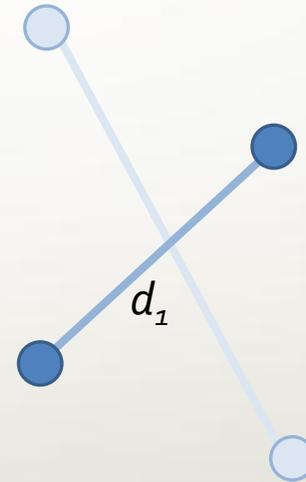
4 Point Congruent Set

- Extract pairs



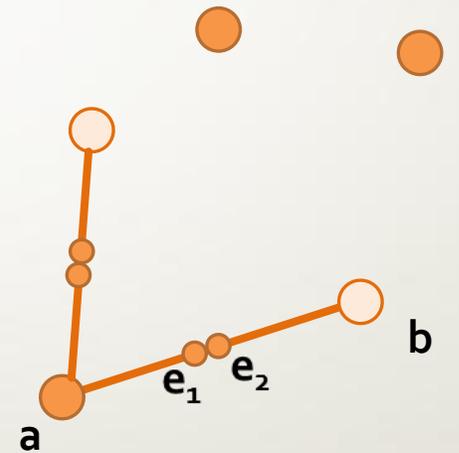
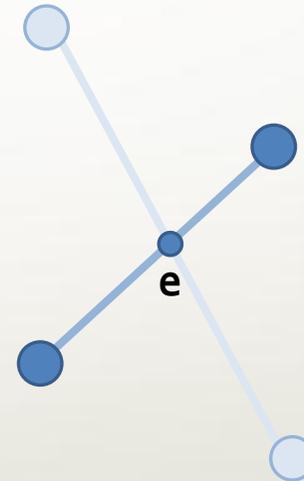
4 Point Congruent Set

- Extract pairs



4 Point Congruent Set

- Extract pairs

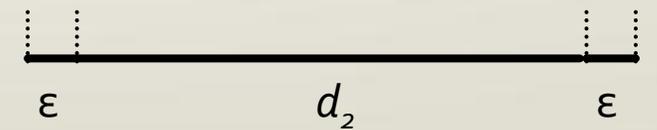
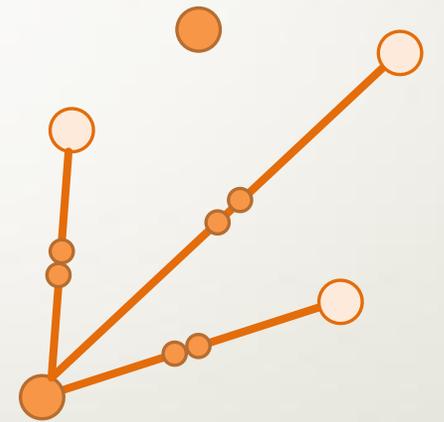
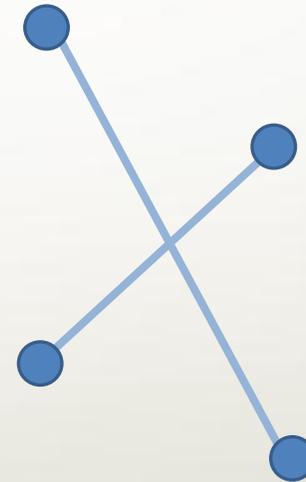


$$\mathbf{e}_1 = \mathbf{a} + r_1(\mathbf{b} - \mathbf{a})$$

$$\mathbf{e}_2 = \mathbf{a} + r_2(\mathbf{b} - \mathbf{a})$$

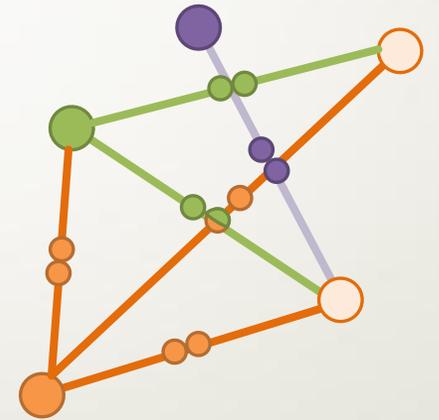
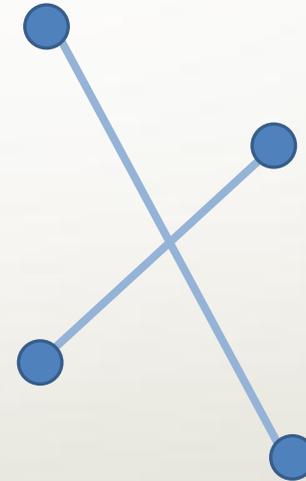
4 Point Congruent Set

- Extract pairs



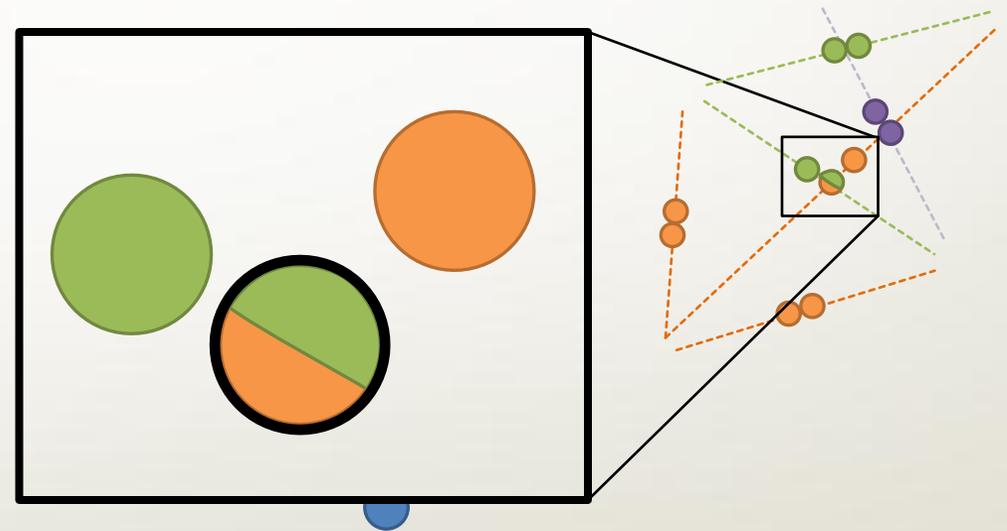
4 Point Congruent Set

- Extract pairs



4 Point Congruent Set

- Extract pairs
- Extract congruent super-set



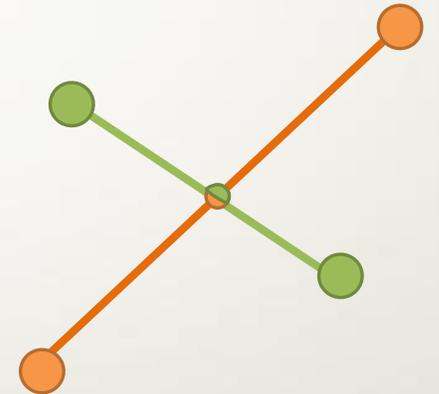
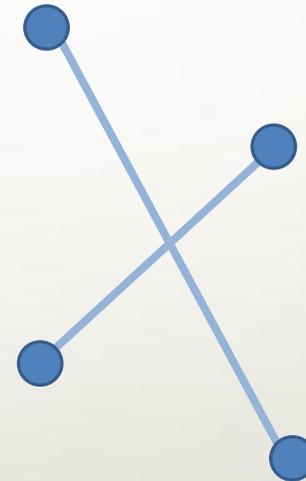
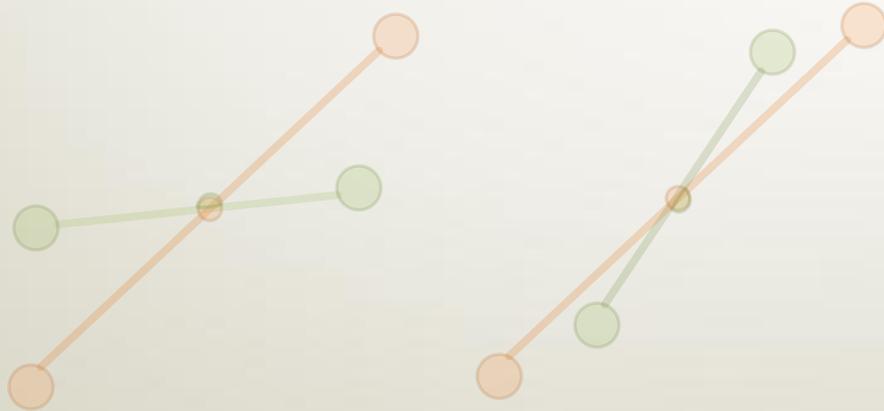
4 Point Congruent Set

- Extract pairs
- Extract congruent super-set

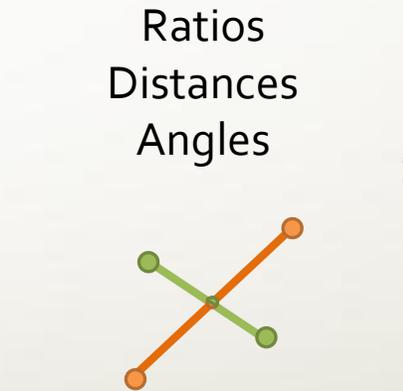
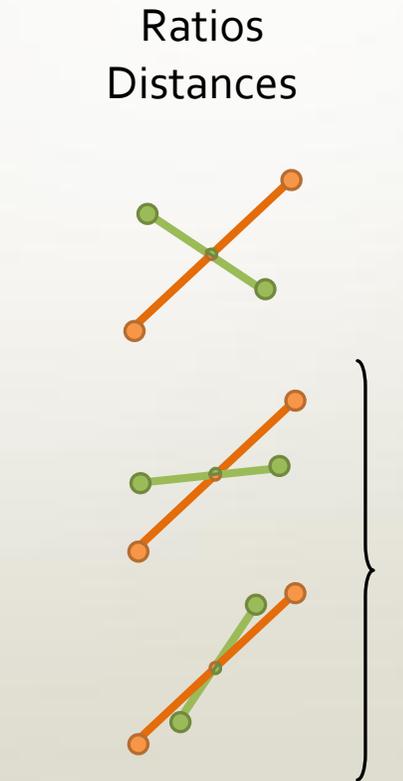
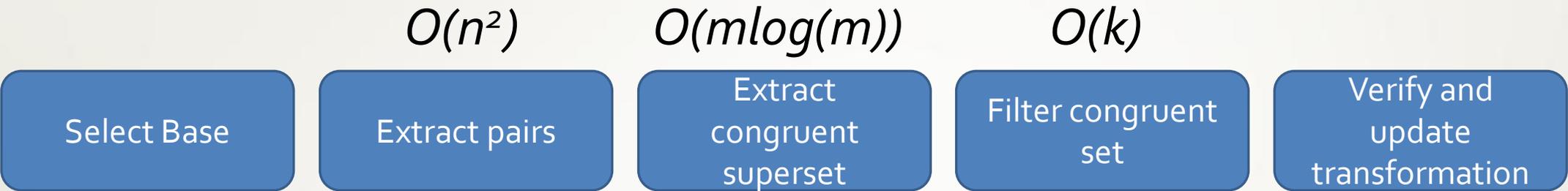


4 Point Congruent Set

- Extract pairs
- Extract congruent super-set
- Filter congruent set

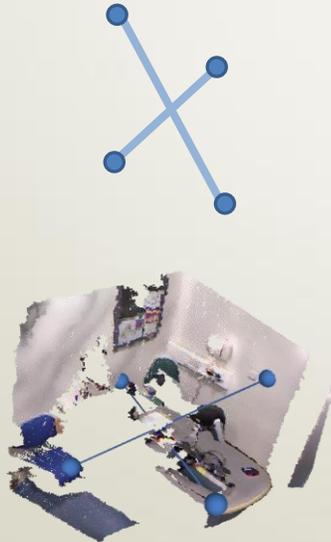
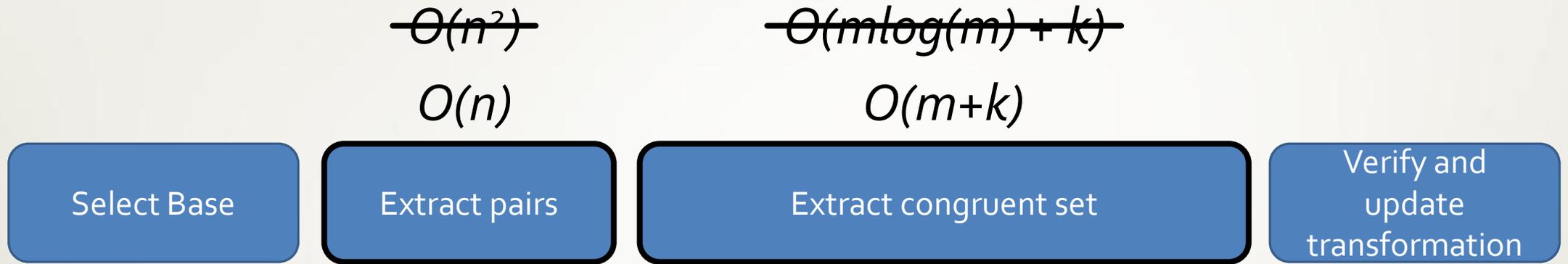


4 Point Congruent Set



n points
 m pairs
 k congruent sets

Super 4PCS



No more false positive



n points
 m pairs
 k congruent sets

Super 4PCS

$$\cancel{O(n^2)}$$

$$O(n)$$

$$\cancel{O(m \log(m) + k)}$$

$$O(m+k)$$

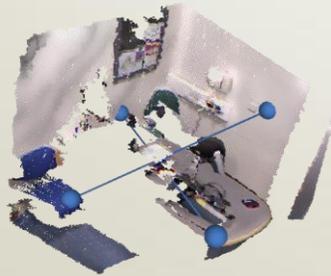
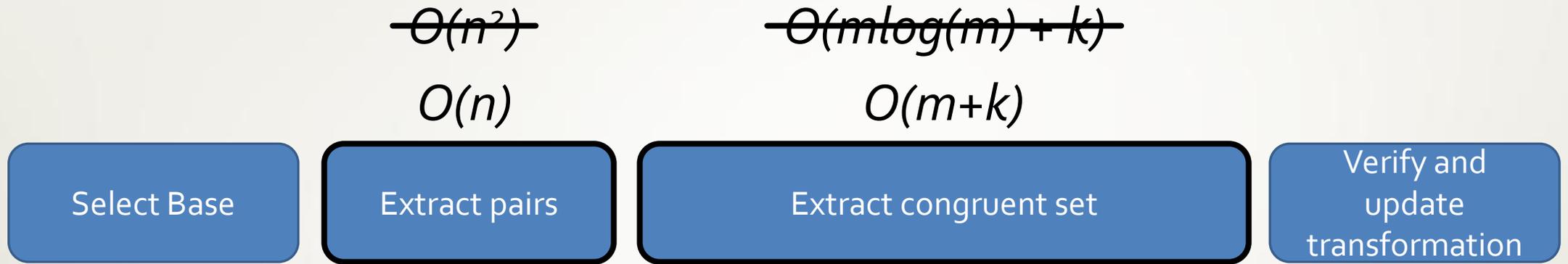
Our smart indexing techniques
produce the **same** congruent set as 4PCS
but in linear time

n points

m pairs

k congruent sets

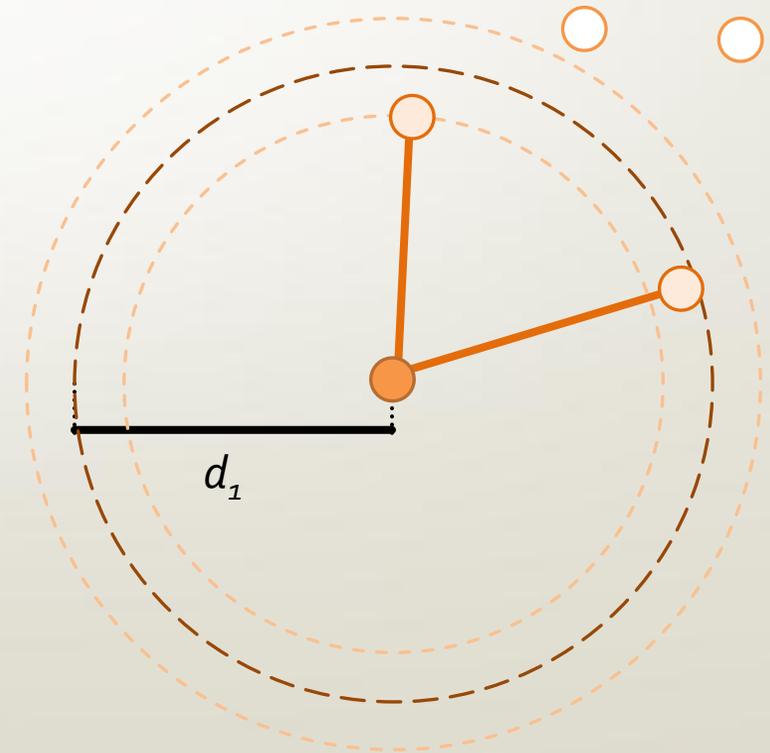
Super 4PCS



n points
 m pairs
 k congruent sets

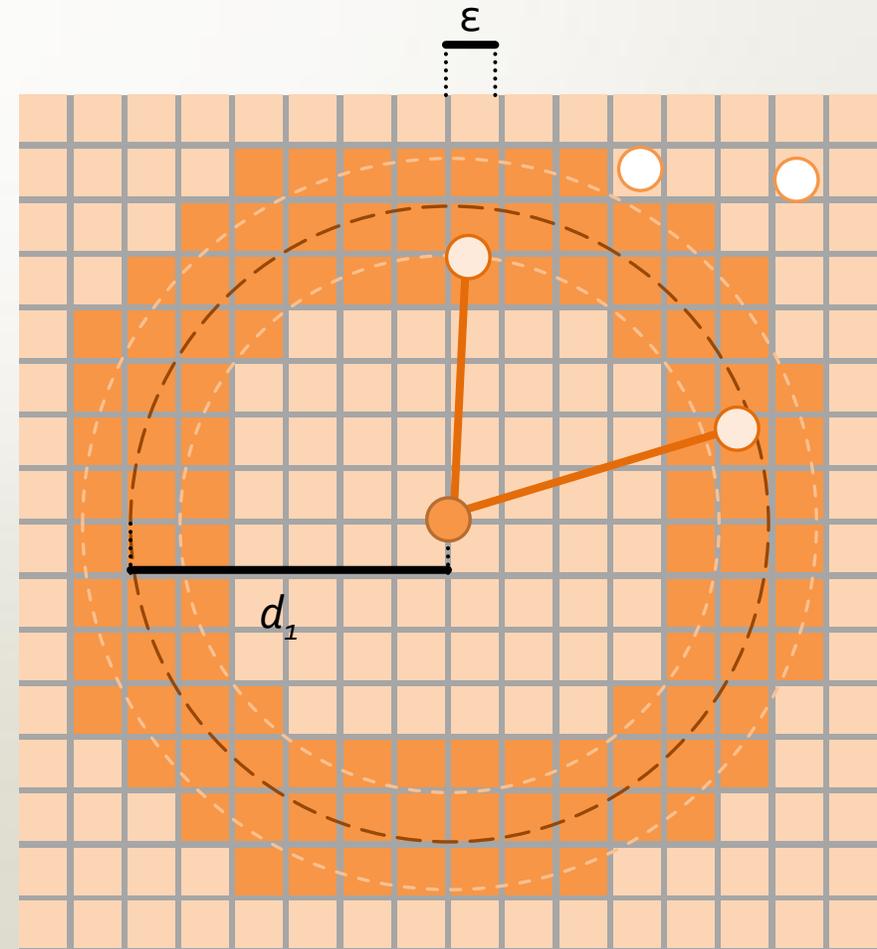
Pair extraction

- Reporting incidences: all valid pairs generated from a sphere



Pair extraction

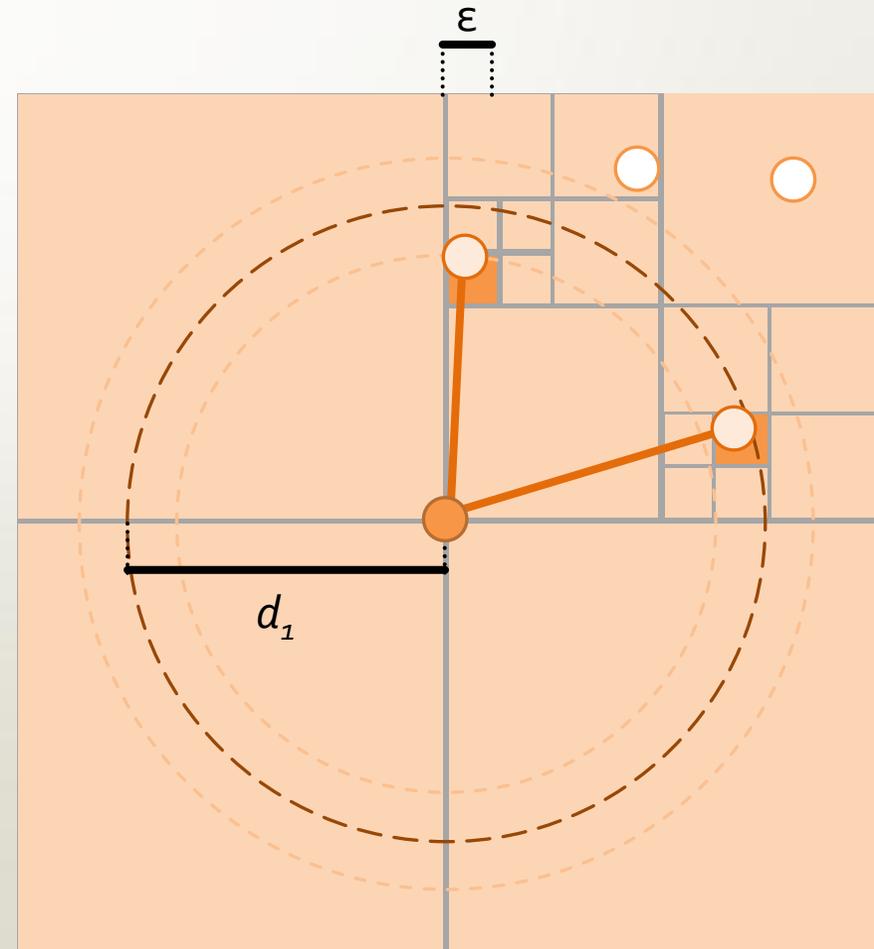
- Reporting incidences using sphere rasterization
 - Complexity depends only on ϵ and d_1



Pair extraction

- Reporting incidences using sphere rasterization
- With an adaptative grid

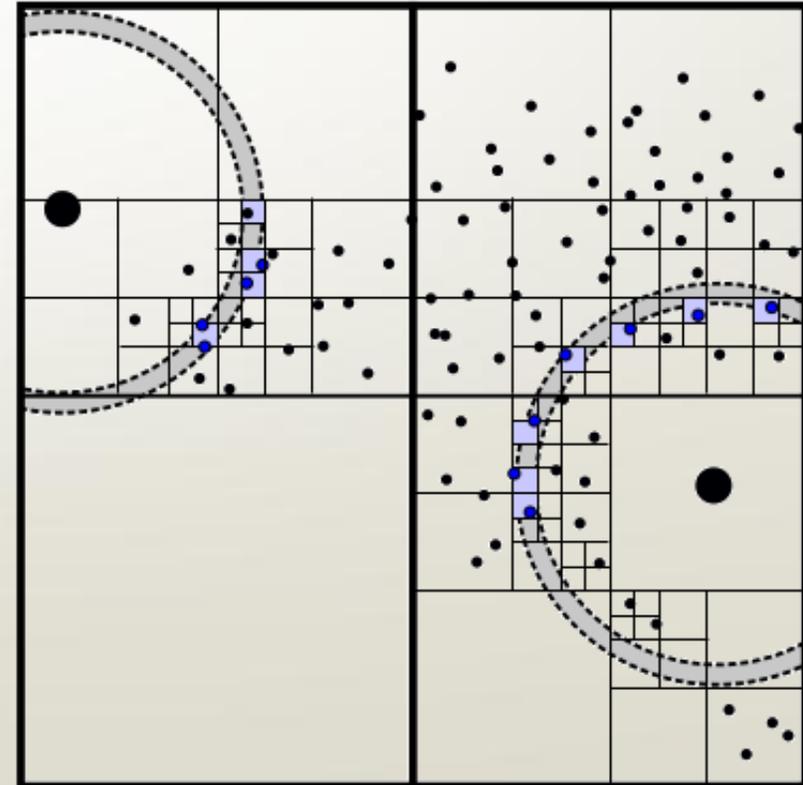
Note: using a pre-computed tree is not optimal



Pair extraction



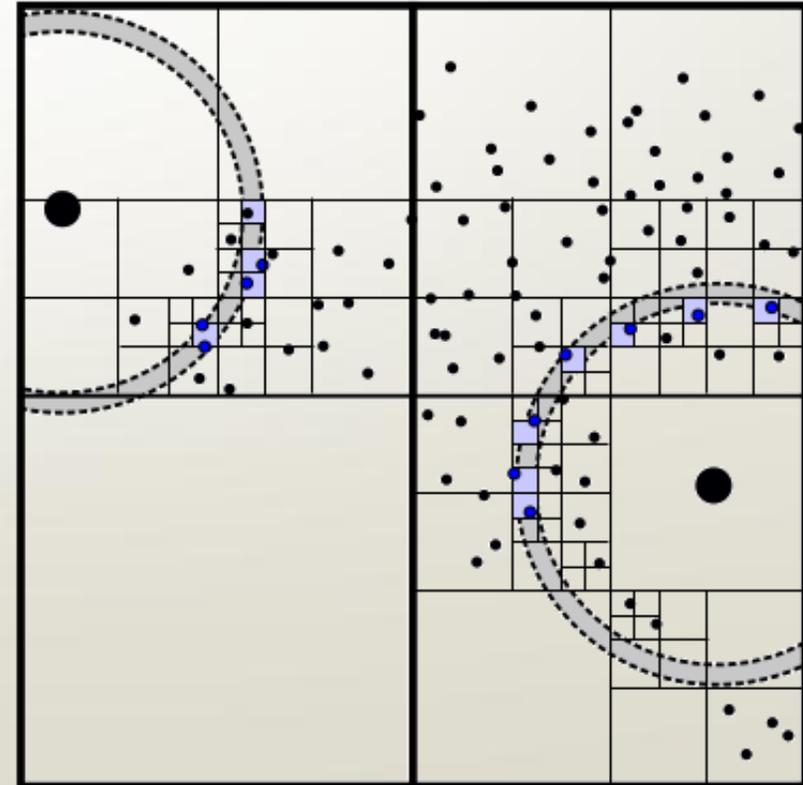
- Reporting incidences using sphere rasterization
- With an adaptative grid
- Simultaneously for all points



Pair extraction



- Reporting incidences using sphere rasterization
- With an adaptative grid
- Simultaneously for all points
- Theoretical complexity: $O(n)$
(see details in the paper)

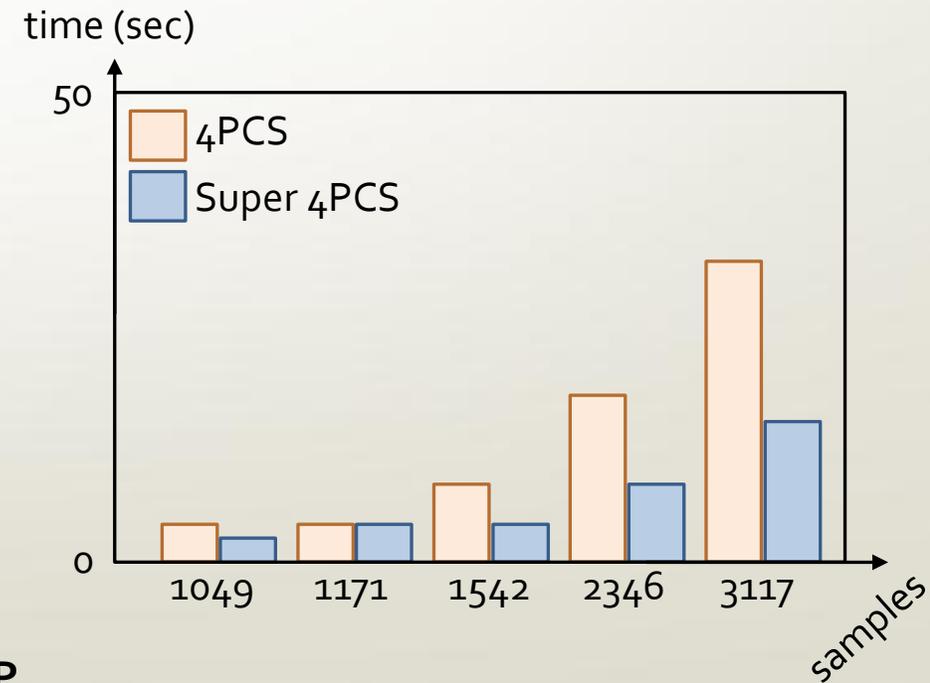


Pair extraction



- Reporting incidences using sphere rasterization
- With an adaptative grid
- Simultaneously for all points

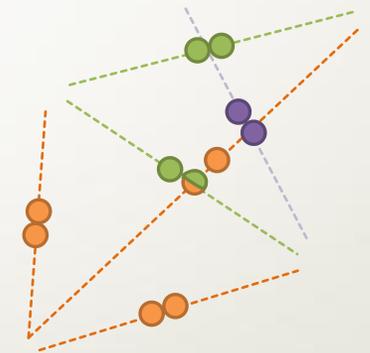
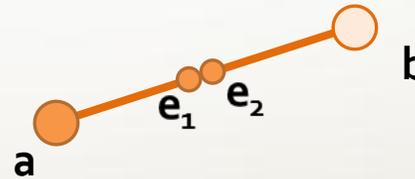
- Theoretical complexity: $O(n)$
- In practice
 - Runtime: linear
 - Minimal memory overhead



4PCS

Congruent set extraction

- Original approach
 - Represent a pair by 2 invariants



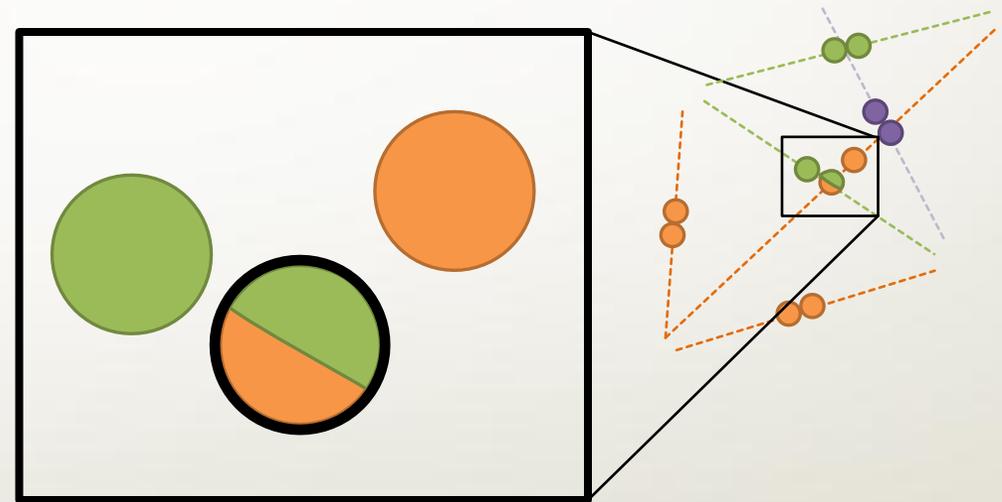
m pairs
 k congruent sets

4PCS

Congruent set extraction



- Original approach
 - Represent a pair by 2 invariants
 - Use **kd-tree** to find closest invariants



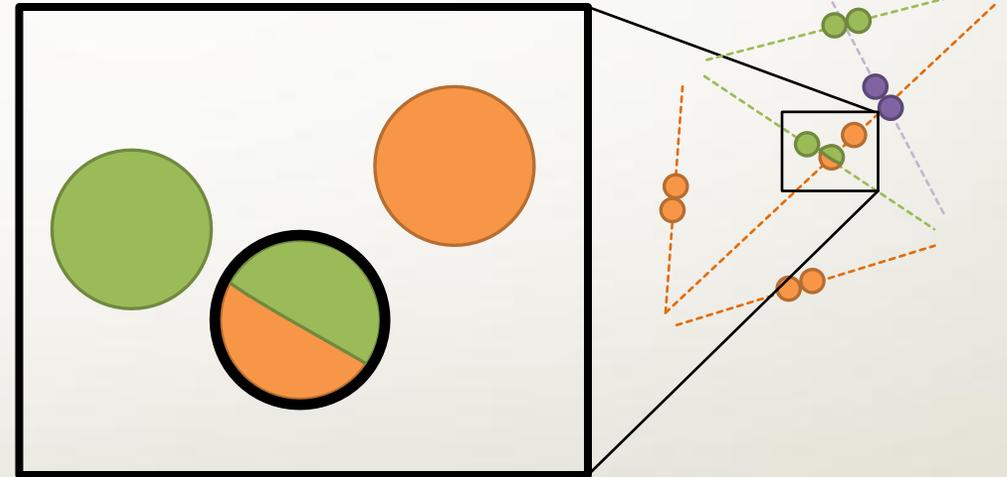
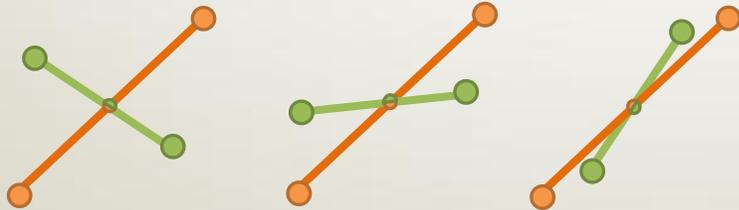
$O(m \log(m))$

m pairs
 k congruent sets

4PCS

Congruent set extraction

- Original approach
 - Represent a pair by 2 invariants
 - Use **kd-tree** to find closest invariants
 - Filter non congruent quadruplets



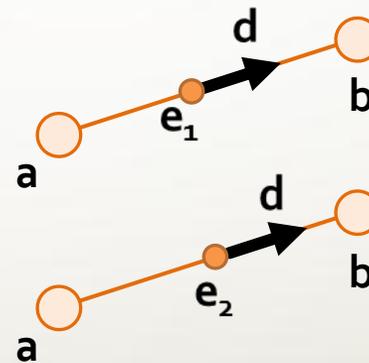
$$O(m \log(m) + k)$$

m pairs
 k congruent sets



Congruent set extraction

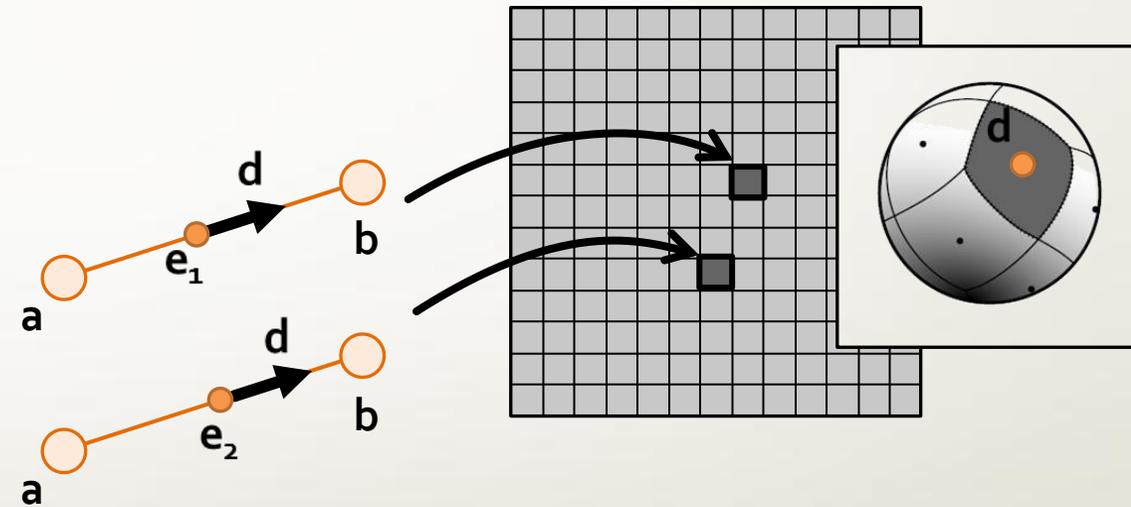
- Efficient indexing
 - Represent pairs as invariant + direction



Congruent set extraction

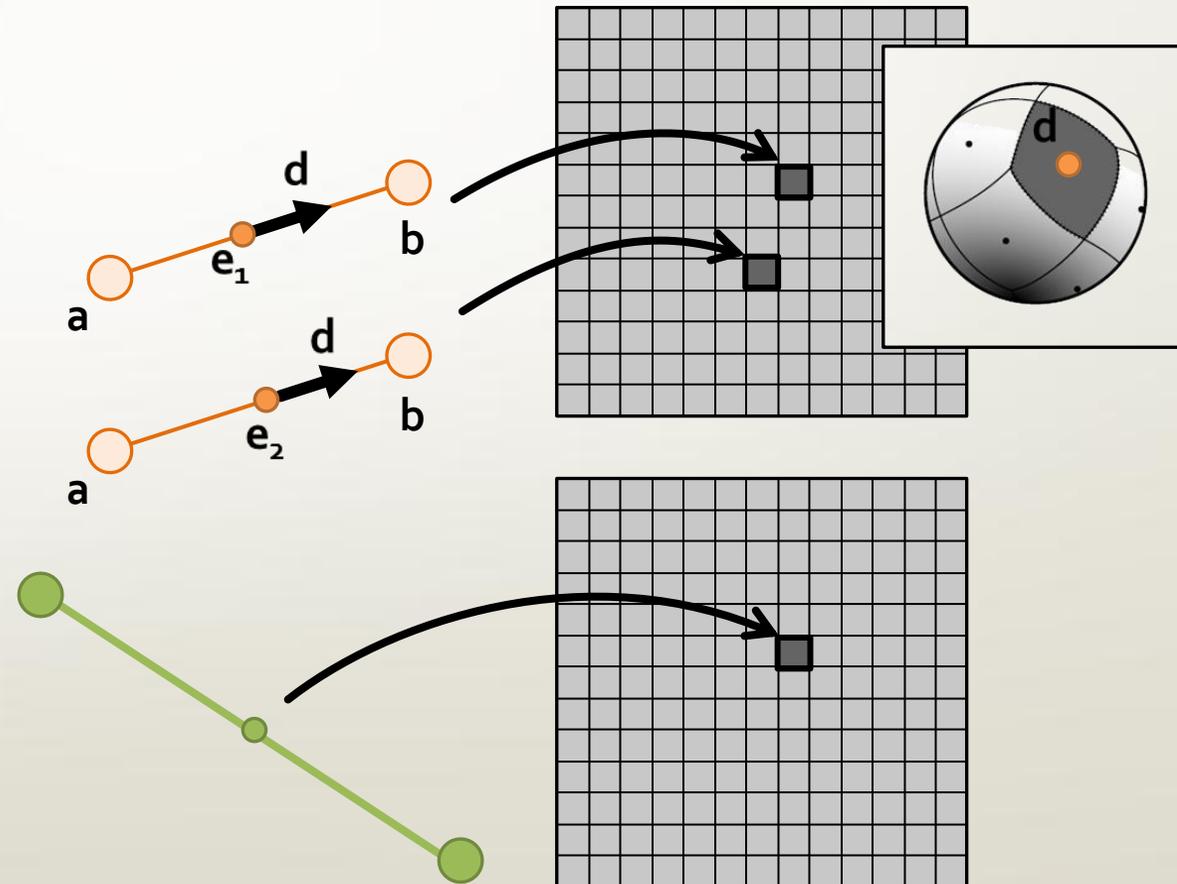


- Efficient indexing
 - Represent pairs as invariant + direction
 - Hash pairs by position and direction



Congruent set extraction

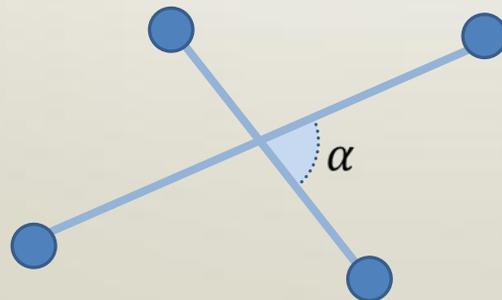
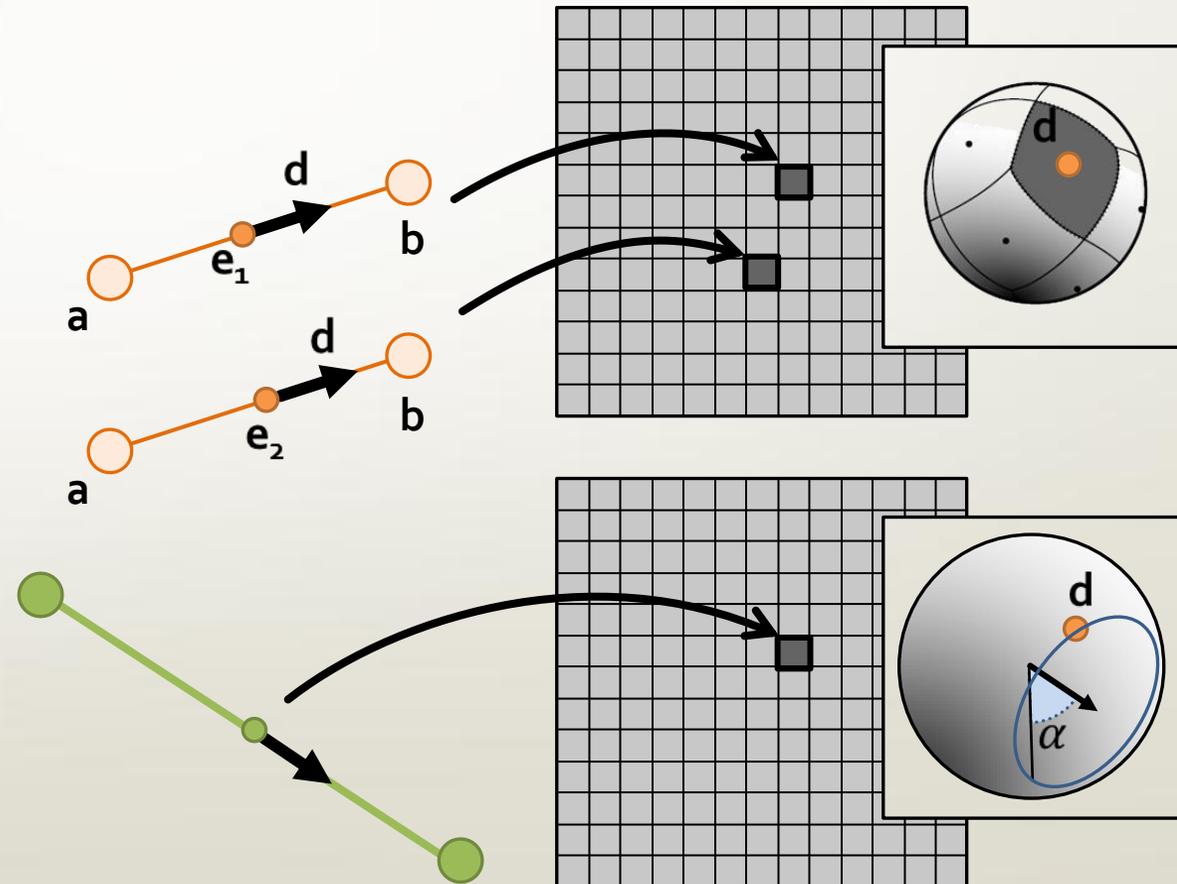
- Efficient indexing
 - Represent pairs as invariant + direction
 - Hash pairs by position and direction
- Query
 - Hash positions (closest invariants)



Congruent set extraction



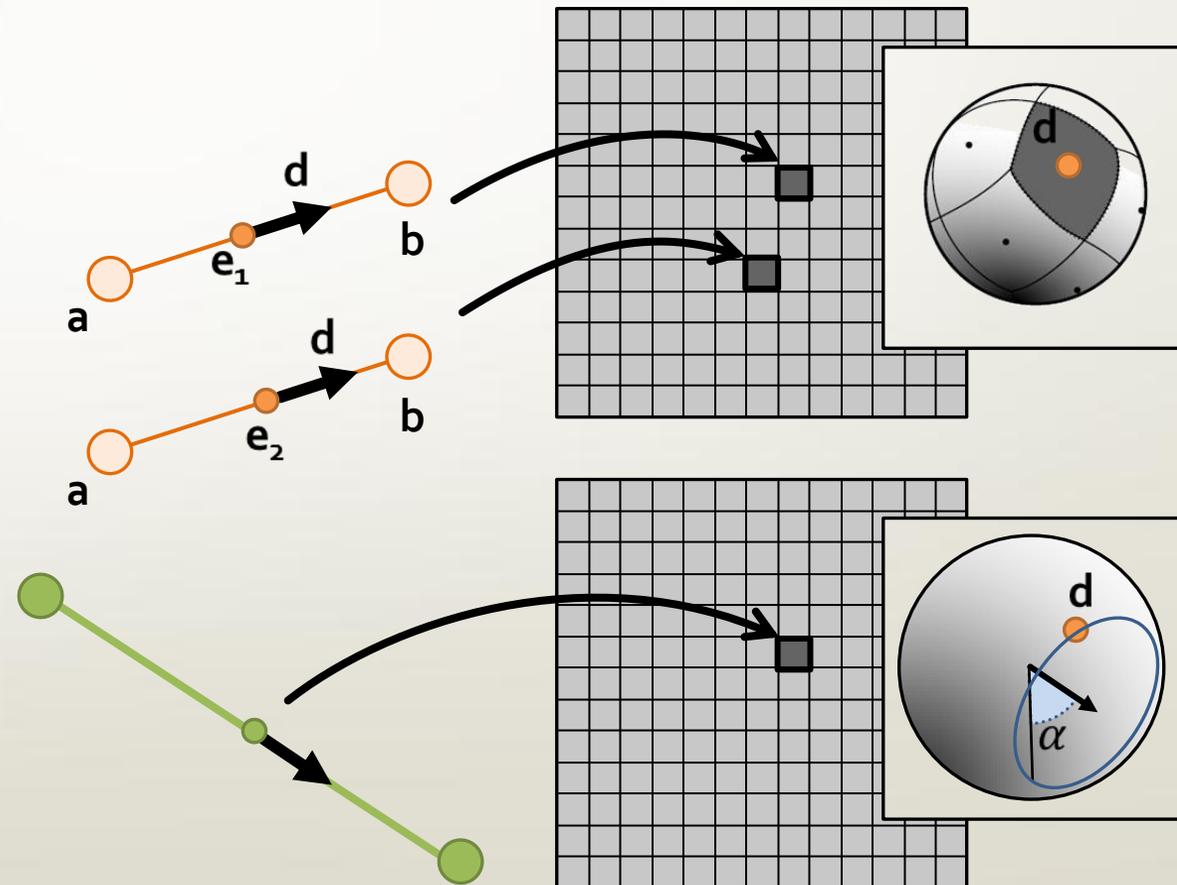
- Efficient indexing
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 - Hash pairs by position and direction
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 - Hash positions (closest invariants)



Congruent set extraction



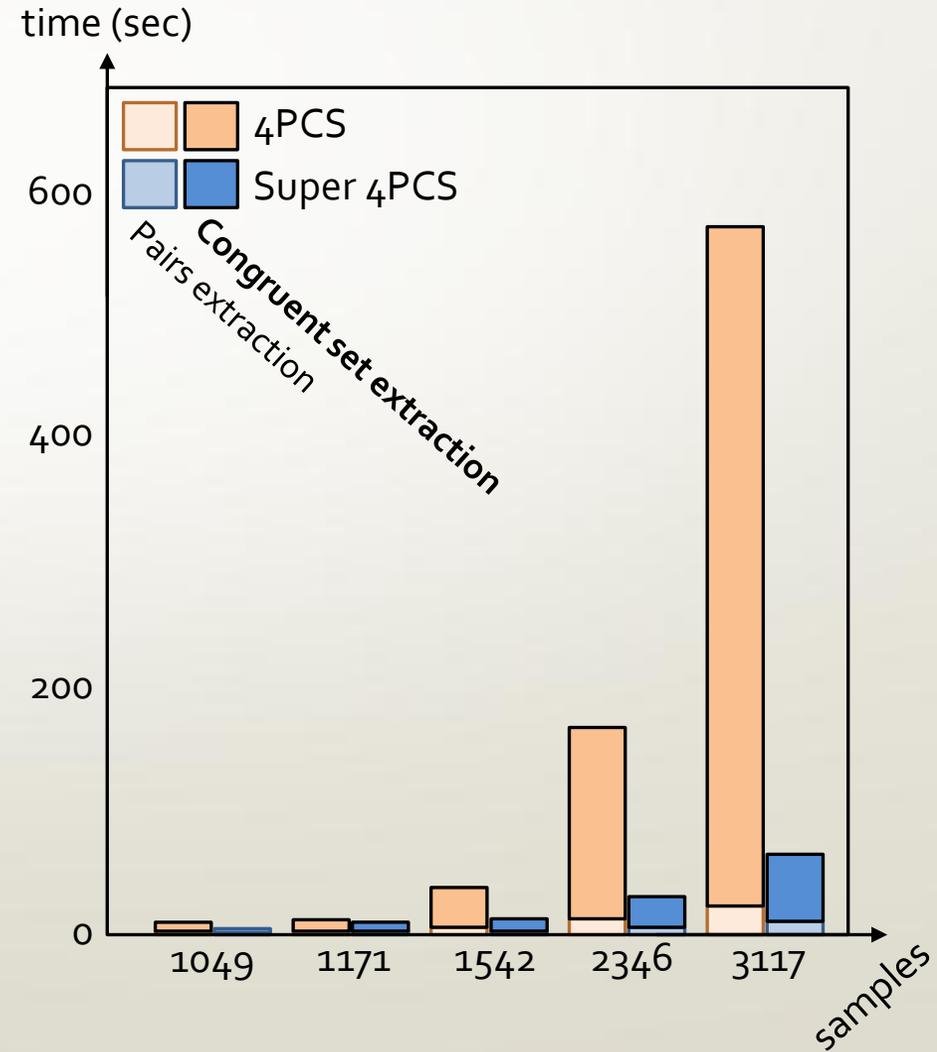
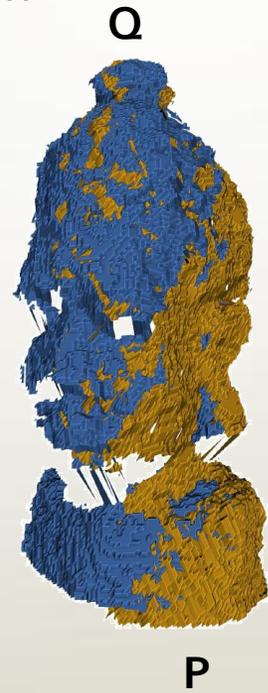
- Efficient indexing
 - Represent pairs as invariant + direction
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 - Theoretical complexity: $O(n)$
(see details in the paper)

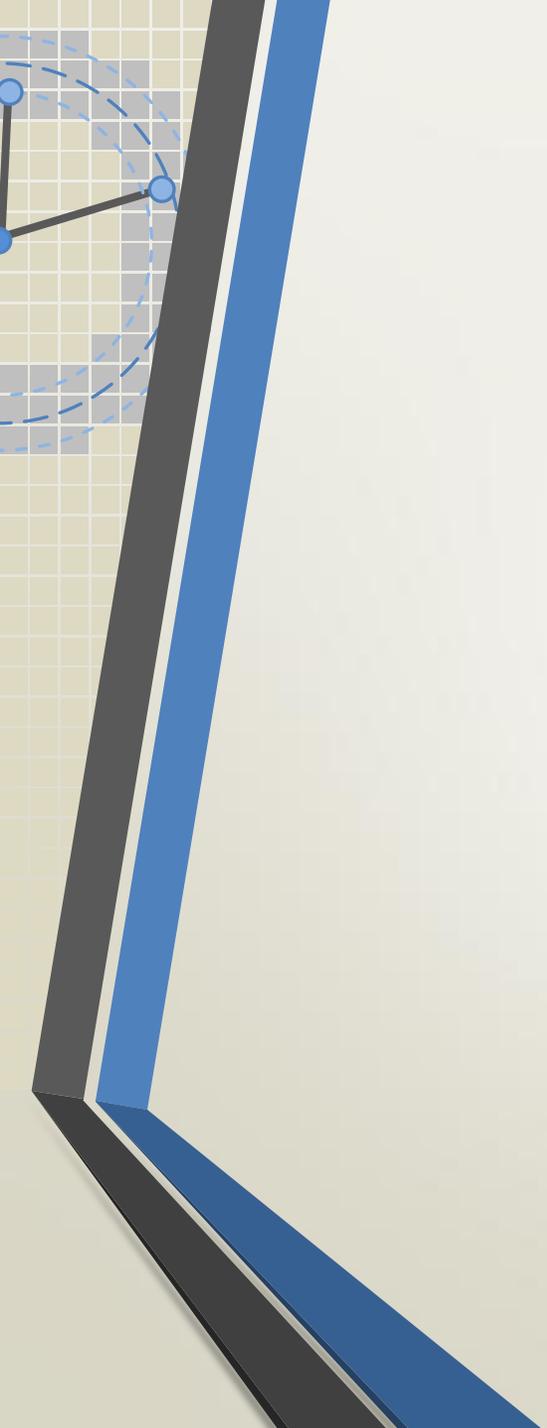


Congruent set extraction



- Efficient indexing
 - Represent pairs as invariant + direction
 - Hash pairs by position and direction
- Query
 - Hash positions (closest invariants)
 - Theoretical complexity: $O(n)$
 - In practice
 - Runtime: linear
 - Memory overhead: similar to kd-tree





Results

Outliers

35k points

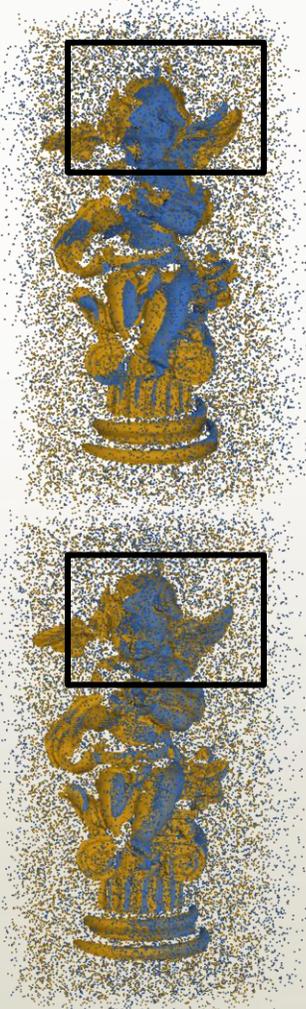


Input

1.4 sec

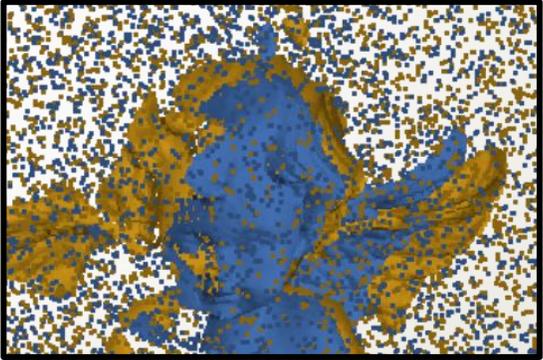


15 sec

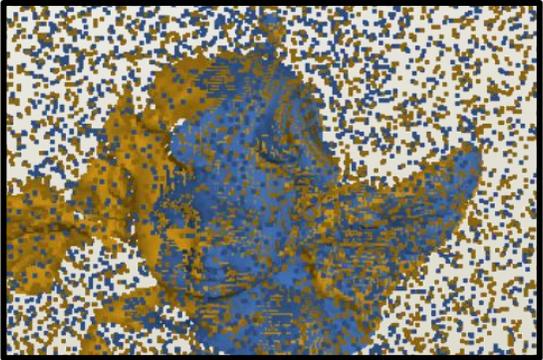


30 sec

Before ICP

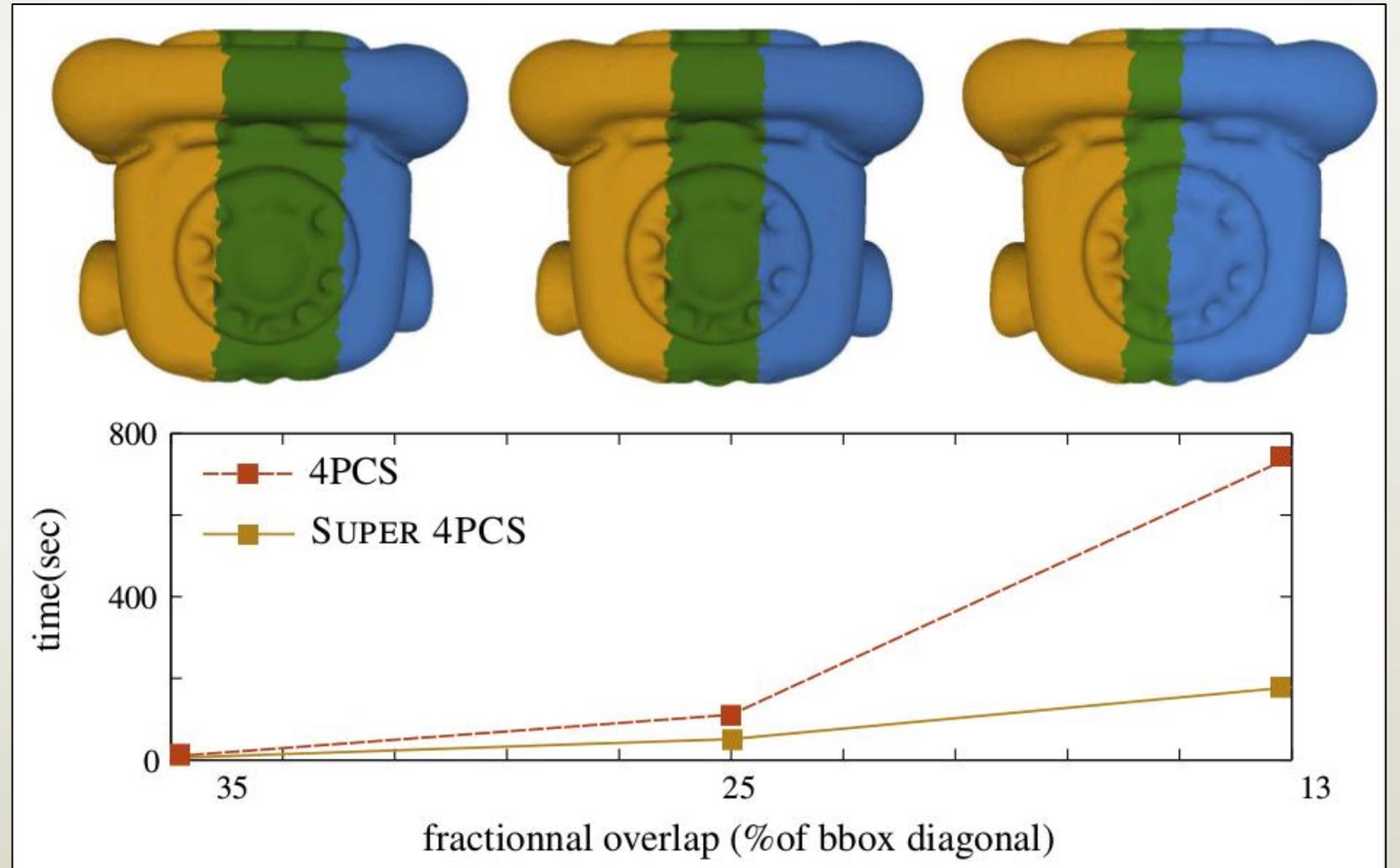


After ICP



Low overlap

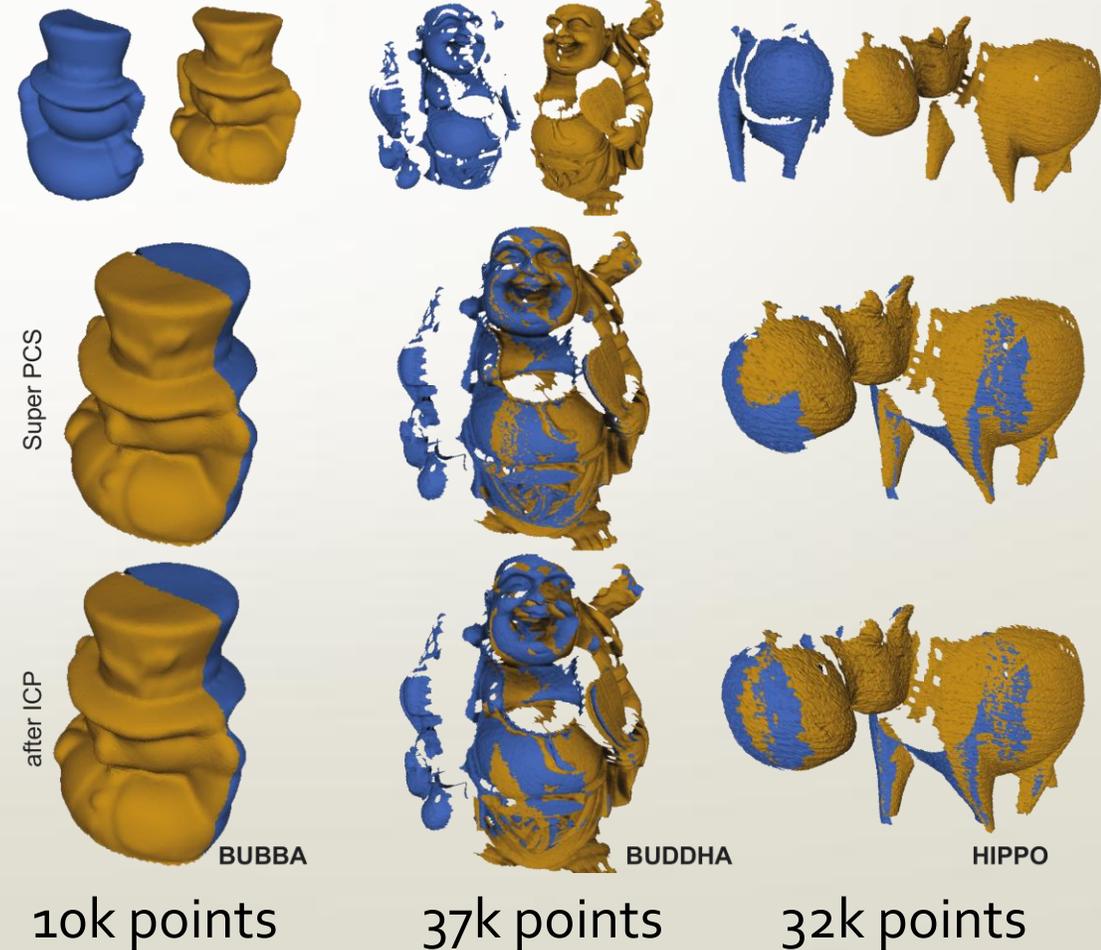
(no ICP)
32k points



Low overlap

- Other examples

Model	Points (x1000)	Overlap (%)	4PCS (in sec)	Super 4PCS (in sec)
Bubba	10	5	5	2.5
Buddha	37	20	63	37
Hippo	32	25	11	0.5



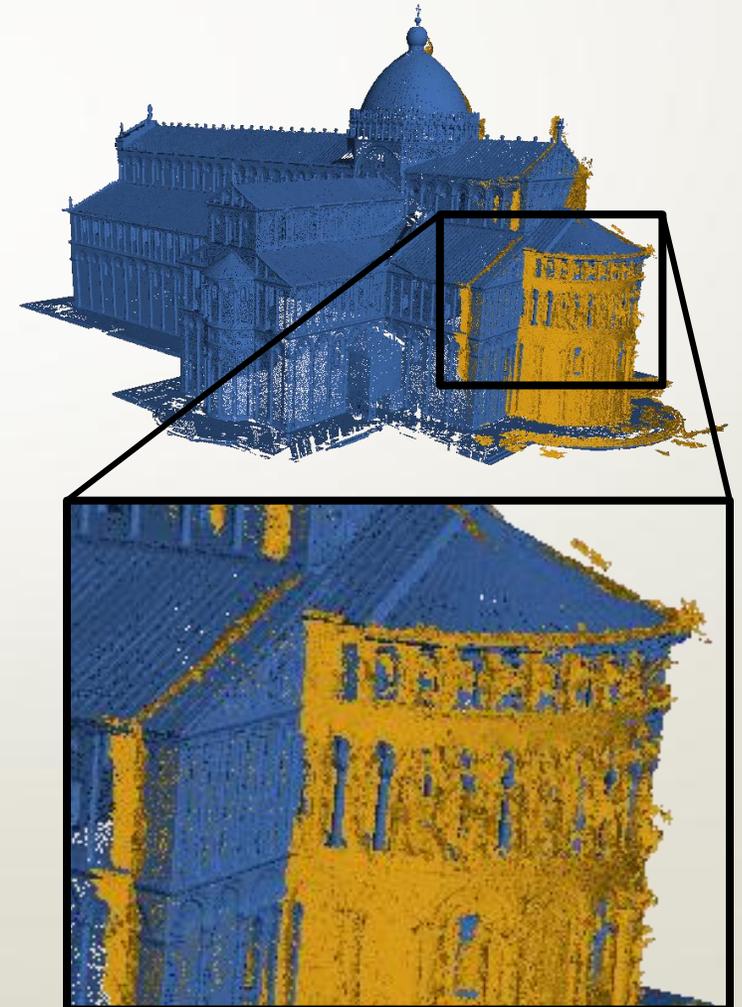
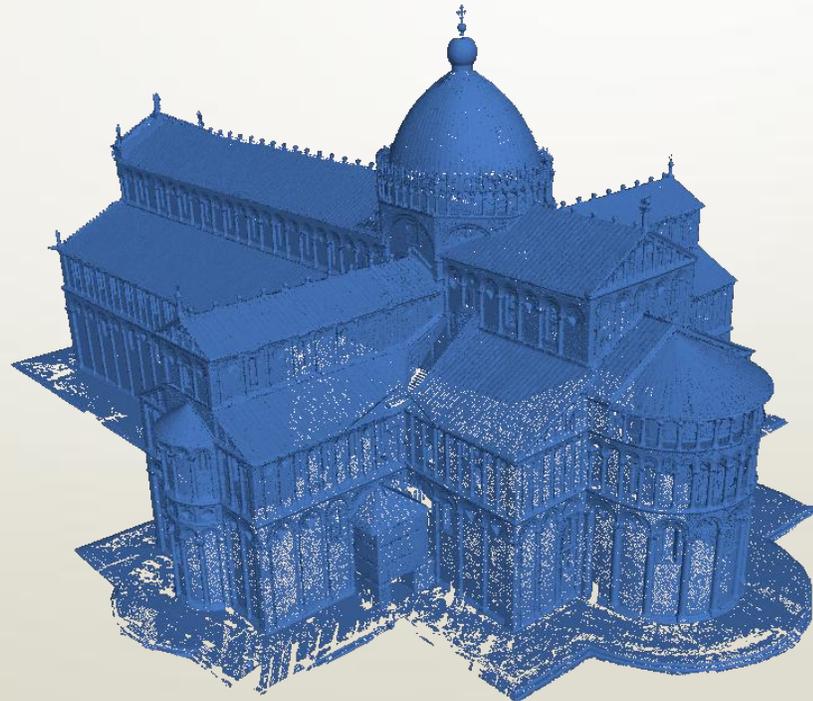
Low overlap + featureless



2 500 points

Multi-modal models

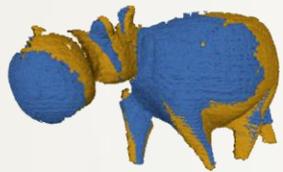
- P: 2.5M points, multiview stereo
- Q: 2.5M points, LIDAR



Range Images alignment



Top: Input
Bottom: initial pose



SUPER 4PCS, no ICP
Top: 0.5 sec
Bottom: 11 sec



ICP
Top: 16 sec
Bottom: 28 sec



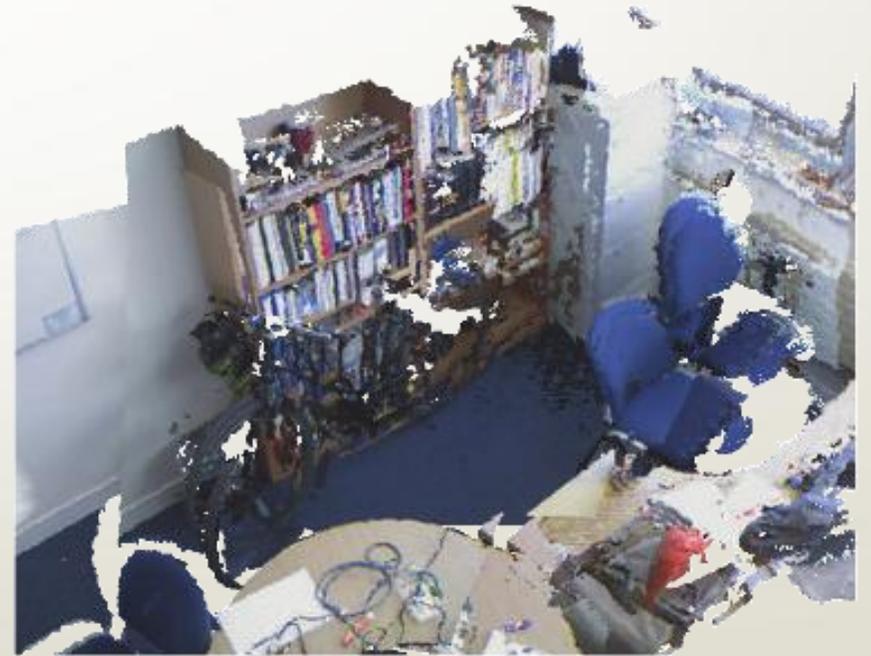
Sparse-ICP ($p = 1$)
Top: 12 sec
Bottom: 43 sec



Sparse-ICP ($p = 0.5$)
Top: 71 sec
Bottom: 60 sec

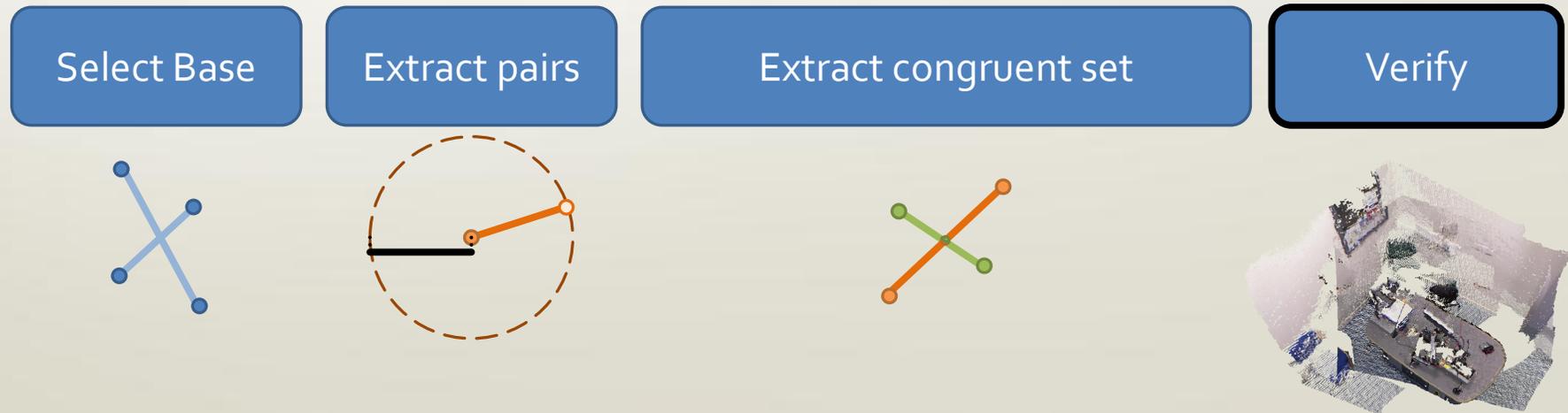
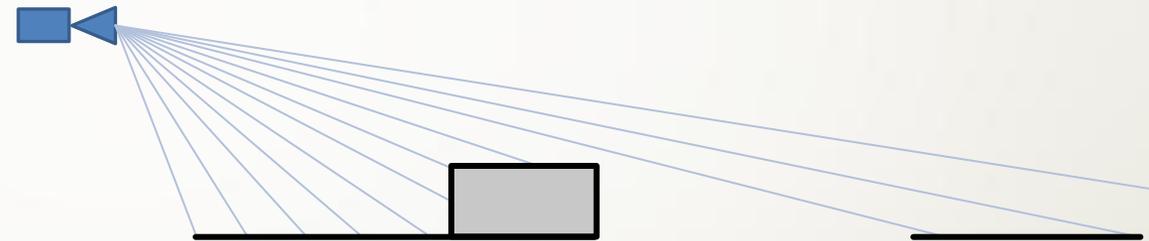
Kinect scans

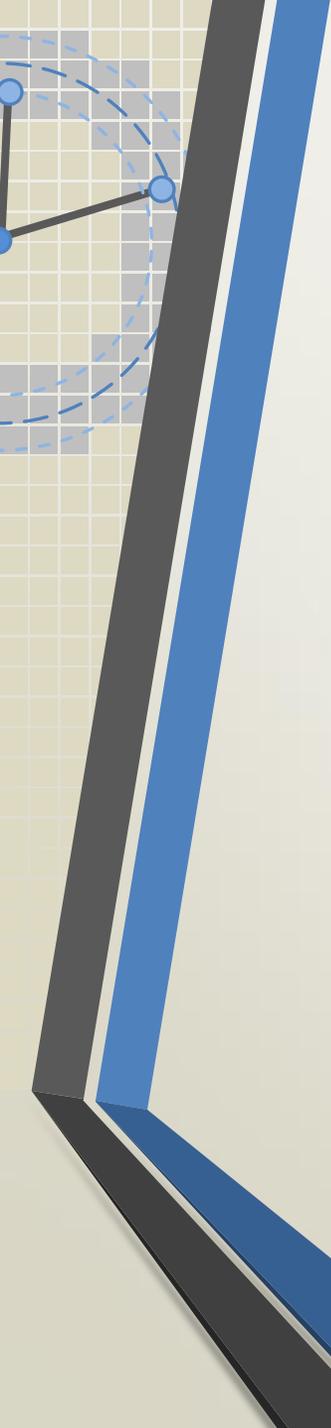
- Chaining pairwise registration (5/6 frames)



Limitations

- Sampling sensitivity
 - Region Of Interest (ROI)
 - Cannot match *between* the points
- Metric





Conclusion

- Global matching algorithm
 - Running in linear time
 - Unstructured point clouds without normals
 - Can be combined with local descriptors
- Future work
 - Real-time using GPGPU programming
 - Alternative to Kinect Fusion

Thank you for your attention

- Super 4PCS
 - Global matching running in linear time

- Acknowledgements

- Feedback&discussion: Duygu Ceylan, Aron Monszpart
- SparseICP data and comparisons: Sofien Bouaziz, Andrea Tagliasacchi
- Pisa dataset: Matteo Dellepiane

- Funding

- Marie Curie Career Integration Grant
- ERC Starting Grant SmartGeometry
- Adobe Research

Code and data goo.gl/uQrhJU



```
File Edit View Search Terminal Help
~/git/super4pcs/codes ./4pcs
Use Super4PCS
Work with 226 points
norm_dist: 5.000000
Initial ICP: 0.061947
Computation time (sec): 0.596068
Score: 0.45132744
0.451327
(Homogeneous) Transformation from input2.obj to input1.obj:
      0.978      -0.171      -0.118      90.768
      0.071      0.800      -0.525      306.750
      0.185      0.564      0.803      93.319
      0.000      0.000      0.000      1.000
saving transform matrix to output_transform.m
Merged object was written to output.obj
~/git/super4pcs/codes
```

github.com/smartgeometry-ucl/Super4PCS

Features:

- C++
- Based on Eigen
- Structures implemented in arbitrary dimensions